

EXERCISES

1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Define $f^{\circ} 0 = f$ and $f^{\circ} n = f \circ f^{\circ} (n-1)$ for $n \geq 1$. Suppose that $f^{\circ} 2 = f^{\circ} 1 \circ f^{\circ} 1$. Prove that $f^{\circ} n = f^{\circ} 1 \circ f^{\circ} (n-1)$ for all $n \geq 1$.

2. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Define $f^{\circ} 0 = f$ and $f^{\circ} n = f \circ f^{\circ} (n-1)$ for $n \geq 1$. Suppose that $f^{\circ} 2 = f^{\circ} 1 \circ f^{\circ} 1$. Prove that $f^{\circ} n = f^{\circ} 1 \circ f^{\circ} (n-1)$ for all $n \geq 1$.

3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Define $f^{\circ} 0 = f$ and $f^{\circ} n = f \circ f^{\circ} (n-1)$ for $n \geq 1$. Suppose that $f^{\circ} 2 = f^{\circ} 1 \circ f^{\circ} 1$. Prove that $f^{\circ} n = f^{\circ} 1 \circ f^{\circ} (n-1)$ for all $n \geq 1$.

4. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Define $f^{\circ} 0 = f$ and $f^{\circ} n = f \circ f^{\circ} (n-1)$ for $n \geq 1$. Suppose that $f^{\circ} 2 = f^{\circ} 1 \circ f^{\circ} 1$. Prove that $f^{\circ} n = f^{\circ} 1 \circ f^{\circ} (n-1)$ for all $n \geq 1$.