

THEORY

1. The first part of the theory discusses the concept of a function and its properties. It covers the domain, codomain, and range of a function, as well as the idea of a mapping between sets. The text explains how a function can be represented by a set of ordered pairs or a graph on a coordinate plane. It also introduces the concept of a composite function and the inverse of a function, where applicable.

2. The second part of the theory focuses on the properties of functions, such as injectivity (one-to-one), surjectivity (onto), and bijectivity (one-to-one correspondence). It discusses how these properties relate to the existence of an inverse function. The text also covers the concept of a continuous function and the Intermediate Value Theorem, which states that if a function is continuous on a closed interval, it takes on every value between its minimum and maximum values.

Property	Definition	Example
Injective	A function f is injective if different elements in the domain map to different elements in the codomain.	$f(x) = 2x$ is injective.
Surjective	A function f is surjective if every element in the codomain is mapped to by at least one element in the domain.	$f(x) = x^2$ is not surjective on \mathbb{R} , but is surjective on \mathbb{R}^+ .
Bijective	A function f is bijective if it is both injective and surjective.	$f(x) = x$ is bijective.
Continuous	A function f is continuous at a point a if the limit of $f(x)$ as x approaches a is equal to $f(a)$.	$f(x) = x^2$ is continuous everywhere.

PROBLEMS

The handwritten solutions for the problems section include:

- Problem 1:** A graph of a function $f(x)$ on the interval $[0, 1]$. The function starts at $(0, 0)$, increases to a peak at $(0.5, 1)$, and then decreases to $(1, 0)$. The graph is labeled with $f(x)$ and the interval $[0, 1]$.
- Problem 2:** A graph of a function $f(x)$ on the interval $[-1, 1]$. The function is a parabola opening downwards, with its vertex at $(0, 1)$ and x-intercepts at $(-1, 0)$ and $(1, 0)$. The graph is labeled with $f(x)$ and the interval $[-1, 1]$.
- Problem 3:** A graph of a function $f(x)$ on the interval $[0, 1]$. The function is a straight line from $(0, 0)$ to $(1, 1)$. The graph is labeled with $f(x)$ and the interval $[0, 1]$.
- Problem 4:** A graph of a function $f(x)$ on the interval $[0, 1]$. The function is a curve that starts at $(0, 0)$, increases to a peak at $(0.5, 1)$, and then decreases to $(1, 0)$. The graph is labeled with $f(x)$ and the interval $[0, 1]$.